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Strong CP, flavor, and twisted split fermions

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Abstract: We present a natural solution to the strong CP problem in the context of split fermions. By assuming CP is spontaneously broken in the bulk, a weak CKM phase is created in the standard model due to a twisting in flavor space of the bulk fermion wavefunctions. But the strong CP phase remains zero, being essentially protected by parity in the bulk and CP on the branes. As always in models of spontaneous CP breaking, radiative corrections to theta bar from the standard model are tiny, but even higher dimension operators are not that dangerous. The twisting phenomenon was recently shown to be generic, and not to interfere with the way that split fermions naturally weaves small numbers into the standard model. It follows that our approach to strong CP is compatible with flavor, and we sketch a comprehensive model. We also look at deconstructed version of this setup which provides a viable 4D model of spontaneous CP breaking which is not in the Nelson-Barr class.

Keywords: Field Theories in Higher Dimensions, Beyond Standard Model, CP violation.

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Contents

1. Introduction

Among the many mysteries of the standard model, the strong CP problem may provide a unique window into new physics. The reason is that although the strong CP phase $\bar{\theta}$ is tiny ($\bar{\theta} \lesssim 10^{-10}$ [1]), it is both UV sensitive *and* technically natural. Other tuned UV sensitive parameters, such as cosmological constant ($\Lambda_{phys} \sim 10^{-120}$ in units of the Planck mass) correspond to relevant operators and get large quantum corrections independent of their initial classical value. Other technically natural parameters, such as the mass of the electron ($m_e \sim 10^{-6}$ in units of the Higgs vev) are protected by a custodial symmetry (a chiral symmetry in the case of m_e), and so the quantum corrections are proportional to the initial value and thus calculably small. The strong CP phase has the peculiar property that the lagrangian possesses no additional symmetry when $\bar{\theta} = 0$ and yet the radiative corrections to $\bar{\theta}$ are still small. In fact, they are so small that from a model-building point of view, the strong CP problem is easy: we can just choose by hand a vacuum where $\bar{\theta} = 0$ at tree level and be done with it. But then we are forgoing a promising opportunity. The important point is that while formally $\bar{\theta}$ is UV sensitive, in practice it acts much like a flavor parameter. And so by exploring how we may get $\bar{\theta}$ to behave exactly like a flavor parameter, we may gain insight into models of both flavor and CP.

Of course, the real reason to associate the strong CP problem with flavor physics is much more obvious: all CP violation, weak and strong, can be encoded in the Yukawa couplings. This is also the reason that the finite corrections to $\bar{\theta}$ are small: CP violation must involve three generations, and hence many Yukawa couplings [2, 3, 4], most of which are small.[1]

So, it is natural to expect a solution to the flavor puzzle, that is a simplification of the hierarchy of fermion masses, to coordinate with a solution for the CP problem, that is the hierarchy between $\theta_{\rm weak} \sim 1$ and $\bar{\theta}$. Moreover, precisely because $\bar{\theta} = 0$ is not protected by a custodial symmetry there *must* be new physics that distinguishes its tree level value from $\theta_{\rm weak}$, and this physics may also distinguish the three generations of fermions. Yet it is remarkable that most of the solutions to the strong CP problem involve physics which is not directly related to the solution of the SM flavor puzzle.

Let us be a little more concrete. Although there is only one reparameterization invariant strong CP phase $\bar{\theta}$, it is useful to write it as:

$$\bar{\theta} = \theta_{\rm QFD} - \theta_{\rm QCD} \,. \tag{1.1}$$

Here, $\theta_{\rm QFD}$ is related to the phases of the quark masses

$$\theta_{\rm QFD} = {\rm Arg}\,{\rm Det}[Y_u Y_d] \tag{1.2}$$

and $\theta_{\rm QCD}$ is the coefficient of the topological $F\tilde{F}$ term in the QCD lagrangian:

$$\mathcal{L} \supset \theta_{\rm QCD} \frac{g_s^2}{32\pi^2} \varepsilon^{\mu\nu\alpha\beta} F^a_{\mu\nu} F^a_{\alpha\beta} \,. \tag{1.3}$$

This separation is artificial: a chiral rotation of the quarks will move the phase between $\theta_{\rm QCD}$ and $\theta_{\rm QFD}$ through of the chiral anomaly. Thus we can always choose $\theta_{\rm QCD} = 0$. But this can be misleading. For example, if we have some flavor model which manipulates the Yukawas so that $\theta_{\rm QFD} = 0$, we can no longer assume $\theta_{\rm QCD} = 0$ by a gauge choice. That is, it is inconsistent to choose $\theta_{\rm QCD} = 0$ and *then* work on $\theta_{\rm QFD}$. We must address the problems together because the chiral anomaly makes $\theta_{\rm QCD}$ and $\theta_{\rm QFD}$ indistinguishable. On the other hand, we can force $\theta_{\rm QCD} = \theta_{\rm QFD} = 0.$ by imposing a symmetry, such as CP, instead of simply rotating the phase into $\theta_{\rm QFD}$. Of course, because CP is not a symmetry of the standard model, it must broken. But if it is broken spontaneously, then the corrections to $\bar{\theta}$ are finite and calculable, and the distinction between $\theta_{\rm QCD}$ and $\theta_{\rm QFD}$ helps us write down operators and estimate their size. In particular, we can address the

[1] $\bar{\theta}$ also gets an divergent logarithmic renormalization at 7-loops, but with a UV cutoff of $\Lambda \sim M_P$ this turns out to be smaller than the finite piece [2].

strong CP problem through the flavor sector. The question becomes: after spontaneous symmetry breaking, why is the combination of Yukawas in $\theta_{\rm QFD}$ (1.2) small while the Jarlskog combination [5]

$$\theta_{\rm weak} = \mathrm{Arg}\,\mathrm{Det}[Y_u Y_d - Y_d Y_u] \tag{1.4}$$

is nearly maximal? This has the gist of flavor questions, such as why is the top Yukawa large while the others are small and hierarchical. So we are led to search for a unified framework to approach the strong CP and flavor problems at the same time.

Of the many proposed solutions to the strong CP problem, some of which we review and compare in section 5, most have no relation to flavor whatsoever. For example, the Peccei-Quinn solution [6] promotes $\bar{\theta}$ to a field, which dynamically relaxes to zero. It patently has nothing to do with flavor. Another possible solution is that $m_u = 0$, but this seems to be disfavored by lattice calculations [7] (although still conceivable, see e.g. [8]). The models that could relate to flavor all seem to involve spontaneous CP breaking, which will be the main subject of this work.

The archetypal spontaneous CP violating theory was designed by Nelson [9], and generalized by Barr [10]. It introduces additional scalars and vectorlike fermions which transform non-trivially under the standard model flavor group. After a scalar get a complex vev which breaks CP, global symmetries ensure that $\bar{\theta} = 0$ to leading order. But the symmetries allow $\theta_{\rm weak}$ to be large.[2] These Nelson-Barr models bear a strong superficial resemblance to Froggatt-Nielson [13] type flavor models. Indeed, in the the Froggatt-Nielsen paradigm, new flavor symmetries are also added to the standard model, along with heavy scalars which break them. The symmetries and breaking are designed to give the right texture to the standard model CKM. However, even though Froggatt-Nielsen type flavor models and the Nelson-Barr type strong CP models have homologous ingredients — they both involve spontaneous symmetry breaking and new heavy scalars — there does not seem to a compelling model unifying the two.[3]

In the current work, instead of trying to merge Nelson-Barr and Froggatt-Nielson, we mix spontaneous CP breaking with the split fermion approach to flavor physics [17]. The idea behind split fermions is to localize the standard model fields at different places in an extra dimension, so that overlap of their gaussian profiles generates hierarchies. Since the claim-to-fame of the split fermion models is that they do not invoke new symmetries, it is hard to imagine how they could distinguish the strong and weak CP phases. However, extra-dimensions do have new symmetries — 5D Lorentz invariance, and locality — which turn out to provide an almost effortless solution of the strong CP problem.

Indeed, if we take a 5D orbifold and have CP broken by a scalar which vanishes on the boundary, where parity is broken, then either P or CP is a symmetry everywhere, as shown in figure 1. Because $\bar{\theta}$ is P and CP odd, the strong CP phase will vanish at tree level, and its radiative corrections are strongly constrained by locality. In addition, generically the fermion zero-mode wavefunctions will rotate in flavor space [18] as they progress from brane

[2] A perhaps minimal implementation of Barr's criteria can be found in [12].

[3] Candidates include [14], which gives the wrong texture and [15], which involves adjoints of $\mathrm{SO}(3)_{\rm flavor}$, but has not been developed into an actual flavor model. [16] provides a brief review.

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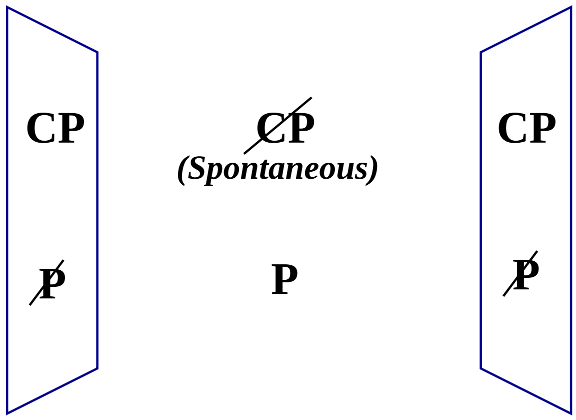

Figure 1: The symmetry breaking structure of our model at leading order.

to brane. This twist can produce large flavor mixing and physical CP violating phases, in particular a large θ_{weak}, but no $\bar{\theta}$. The purpose of paper is to explore the details of this mechanism.

In section 2 we review some features of fermions in five dimensions, including discrete symmetries, splitting and twisting. Section 3 combines these ingredients into a simple model which shows how twisting can address the strong CP problem. This, without extra ingredients, sets $\bar{\theta} = 0$ at leading order and yields $\bar{\theta} \sim \mathcal{O}(10^{-7})$ if we include the strongest higher dimension operators. We then show how some non-abelian flavor symmetries can be added to reduce $\bar{\theta} < 10^{-12}$, which is well in line with current bounds. Some other ways in which the dangerous operators can be suppressed are also discussed. We then show how the setup is compatible with the split-fermions mechanism for generating flavor hierarchies. Section 4 is devoted to the deconstruction of these models, where we show the analog of twisting in a purely four-dimensional context. Although formally the deconstruction works, the 4D version is much messier than the 5D one, basically because parity is badly broken. In section 5 we show why the use of parity in extra dimensions is more natural than in left-right symmetric models. Finally, in section 6 we elaborate our point-of-view, and conclude.

2. 5D fermions, splitting and twisting

The models we will discuss begin in five dimensions on $M_4 \times S_1$ with a set of bulk fermions Ψ^i and scalars Φ^a, and perhaps some other fields living on branes. We will assume CP is an exact symmetry at high energy, and is spontaneously broken by the condensation of at least one of the bulk scalars $\langle\Phi\rangle \neq 0$. A scalar with a position dependent vev can be used to generate fermion hierarchies, and we will see that the same scalar can be used for both purposes.

In 5D, fermions are vectorlike, and can be written in terms of chiral fermions as $\Psi = \Psi_L + \Psi_R$, where $\gamma_5\Psi_L = \Psi_L$ and $\gamma_5\Psi_R = -\Psi_R$. A 5D Lorentz invariant lagrangian is automatically invariant under 5D parity, which interchanges L and R. In particular, a generic Yukawa interaction has the form

$$\mathcal{L}_{5D} = h_{ij}\Phi\bar{\Psi}_i\Psi_j + \text{h.c.} = \left(h_{ij}\Phi + h^\dagger_{ij}\Phi^*\right)\left(\bar{\Psi}_{Li}\Psi_{Rj} + \bar{\Psi}_{Ri}\Psi_{Lj}\right). \tag{2.1}$$

Here, i and j are flavor indices. After Φ gets a vev, the effective mass matrix for the fermions is:

$$M_{ij} = h_{ij}\langle\Phi\rangle + h^\dagger_{ij}\langle\Phi^*\rangle\,. \tag{2.2}$$

Note that this matrix is hermitean, even if the Yukawas h_{ij} are complex. We emphasize that this is guaranteed by parity which is part of 5D Lorentz invariance. It is also very different from the way parity works in left-right symmetric models, which we review in section 5.

In order to have a chiral theory, we need to turn our S_1 into an orbifold by modding out by a Z_2. The Z_2 acts on fermions as $\Psi(z) \to \gamma_5\Psi(-z)$ and scalars as $\Phi(z) \to \pm\Phi(-z)$. Note that this symmetry forbids a bulk mass term of the form $m\bar{\Psi}\Psi$, since it sends $\bar{\Psi} \to -\bar{\Psi}\gamma_5$, but allows the Yukawa interactions in (2.1) if the Φ is Z_2 odd ($\Phi(z) \to -\Phi(-z)$). Such a Φ must vanish at the orbifold fixed points and thus its vev will be position dependent [19, 20]. When we orbifold, by identifying a field with its Z_2 image, half the fermion zero modes, say the right-handed ones, are projected out.

Following from the 5D lagrangian,

$$\mathcal{L} = \bar{\Psi}_i\left(i\partial\!\!\!/ - \gamma^5\partial_z\right)\Psi_i + M_{ij}(z)\bar{\Psi}^i\Psi^j \tag{2.3}$$

the left-handed zero mode profiles are determined by

$$\left[\partial_z\delta_{ij} - M_{ij}(z)\right]\psi^j_{L\alpha} = 0\,. \tag{2.4}$$

Note that for a generic z-dependent matrix $M_{ij}(z)$, this is a set of three (for three flavors) coupled differential equations. The spectator index α distinguishes the three independent solutions and will become the flavor index in the effective 4D theory. The form of M and its z-dependence, produces both splitting, which can establish flavor hierarchies, and twisting, which we will use here for strong CP.

Let us write solutions to (2.4) in terms of an evolution operator K_{ij}:

$$\psi^j_L(z) = K_{ij}(z)\psi^i_L(0)\,, \tag{2.5}$$

where we take $\psi^i_L(0)$ to be an eigenstate of $M_{ij}(0)$. It follows that K_{ij} can be written as a Dyson series:

$$K_{ij}(z) = P\left\{e^{\int_0^z M(z')\mathrm{d}z'}\right\} = \lim_{N\to\infty}\prod_n^N\left[1 + (z_{n+1} - z_n)M(z_n)\right], \tag{2.6}$$

where $P\{\}$ denotes the path-ordered product. An important point is that each term in this product is an hermitean matrix, and so

$$\mathrm{Det}[K_{ij}(z)] \in \mathbb{R}, \qquad \text{for each } z\,. \tag{2.7}$$

There is a subtlety here about whether ψ_L^i are orthonormal, and if they are not whether their diagonalization will introduce a phase into K. In appendix A we show this is not a problem.

In the case that M_{ij} can be diagonalized with a z-independent flavor rotation, the path ordering is trivial. So K_{ij} is diagonalized too. Then one can see from the exponential form in (2.6) that the zero mode wavefunctions will be localized. Because the different flavors may be split, that is, localized at different points, their overlaps can be hierarchical. This is the split fermion solution to the flavor problem.

In general, however, M_{ij} will not be diagonalizable at every z by a global flavor rotation. In fact [18], if

$$\big[M(z), M'(z)\big] \neq 0\,, \qquad \text{for some } z\,. \tag{2.8}$$

then a zero mode fermion at the boundary $z = \pi$ will differ from its value at $z = 0$ by a twist $K_{ij}(\pi)$. Note that even if $M_{ij}(z)$ vanishes at 0 and π, which it will because of the boundary conditions on Φ, the last expression in (2.6) shows that there will still be non-trivial twisting because of the intermediate values of M_{ij}. As we will show in the next section, this is exactly what we need for the CP problem: because K is a complex matrix with real determinant, it can add a weak CP phase into the Yukawas without adding a strong one.

To be more explicit, CP works in 5D just like in 4D: it acts on fields by hermitean conjugation. That is, $\Phi \to \Phi^\star$ and $\Psi \to \bar{\Psi}$ (and so $\psi_L \to \bar{\psi}_L$ and $\psi_R \to \bar{\psi}_R$). Unlike parity, CP invariance is not an automatic consequence of 5D Lorentz invariance, but we may impose it. Then, like in 4D, because lagrangians are already hermitean, CP forces all the c-numbers to be real. In particular, the Yukawa couplings h_{ij} in (2.2) must be real.

As usual, CP is only defined up to a phase. So saying CP invariance is equivalent to couplings being real is just a convention.[4] Moreover, we are also free to employ a global flavor rotation to choose our fermion basis, which should not affect the CP properties of the theory. (This is why it is important to use the basis independent definitions of θ_{weak} and θ_{QFD} in (1.4) and (1.2).) In the current context, however, there is an additional subtlety. Even if we cannot use a global flavor rotation to choose an untwisted basis, that is one where $M_{ij}(z)$ is diagonal at each z, if we can choose a basis so that K_{ij} is real, the twist cannot mediate violations of CP. The corresponding basis independent condition on $M_{ij}(z)$ is

$$\operatorname{Arg}\operatorname{Det}[M(z), M'(z)] \neq 0\,, \qquad \text{for some } z\,. \tag{2.9}$$

This will be the necessary condition for the twist to mediate CP violation. Note that this is a stronger condition than that for a twist (2.8), but is still generically satisfied, as we will see in the model in the next section.

3. A simple model for strong CP

We begin with a simple setup which addresses the strong CP problem. We will find that

[4] There is an interesting model [15] of strong CP which uses this ambiguity to show that one can have "real CP violation."

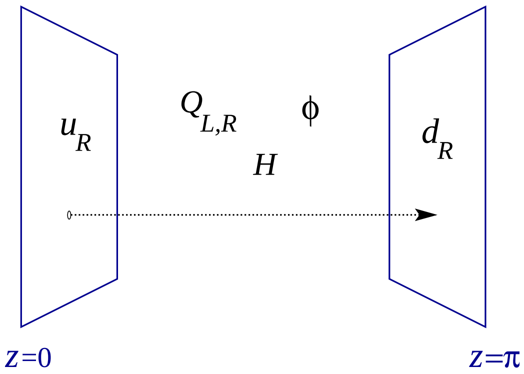

Figure 2: Particle content of the model.

with the right allocation of fields in the bulk and on the boundary, $\bar{\theta}$ is naturally zero, but $\theta_{\rm weak}$ is generically large. As always, the finite radiative corrections to $\bar{\theta}$ are small. So we go on to consider the contribution form higher dimensional operators, which cannot be neglected in the non-renormalizable 5D theory. Although we do not give a detailed flavor model, we also show how our flavor hierarchies can generated within our setup.

3.1 Tree level solution

As discussed in the previous section, the scene takes place on an orbifold S^1/Z_2. We assume CP is a symmetry at high energy, spontaneously broken by a bulk scalar Φ. The SU(2) quark doublets of the SM, Q^i, live in the bulk while the SU(2) singlet quarks, u_R^α and d_R^α, are allocated to opposite fixed points:[5] u_R^α at $z=0$ and d_R^α at $z=\pi$. The right handed component of the bulk fermions Q_R^i are odd under the orbifold Z_2 and thus the right handed zero modes are projected out of the theory. The CP breaking scalar Φ is a standard model singlet but odd under Z_2, so that it can have Yukawa couplings to the fermions. The SM Higgs H lives in the bulk. The setup is summarized in figure 2.

In addition to the kinetic terms, the lagrangian of the model, written in chiral notation, contains Yukawa interactions and a scalar potential:

$$\begin{aligned}\mathcal{L} \supset & \int d^5x \,\left[(h_{ij}\Phi + h_{ij}^T\Phi^*)(\bar{Q}_{Li}Q_{Rj} + \bar{Q}_{Ri}Q_{Lj}) + V_\Phi\right] + \\ & + \int d^4x \left[Y^u_{i\alpha} H \bar{Q}_{Li} u_{R\alpha}\Big|_{z=0} + Y^d_{i\alpha}\tilde{H}\bar{Q}_{Li}d_{R\alpha}\Big|_{z=\pi} + \text{h.c.}\right]. \end{aligned} \tag{3.1}$$

For clarity, we have used i and j to label the three generations of bulk fields and α to label fields on the branes. The Yukawa couplings Y^u, Y^d and h are all real because we assume CP is a good symmetry.

Because the scalar Φ is odd under the orbifold Z_2, its vev must vanish at both fixed points. However, it may have a non-trivial profile if we include a potential [21]. Moreover,

[5] The main difference between our setup and some others [18, 21] is that we localize some of the quarks on branes in 5D. This introduces local anomalies, which can be canceled by adding a parity violating Chern-Simons term in the bulk [22]. However, the easiest way to see that local anomalies and bulk parity violation are fictitious is to think of the singlet quarks as dynamically localized bulk fields.

it is reasonable to give its real and imaginary parts different potentials:

$$V_\Phi = \lambda_R[(\Phi + \Phi^*)^2 - \mu_R^2]^2 + \lambda_I[(\Phi - \Phi^*)^2 - \mu_I^2]^2 \tag{3.2}$$

so that the phase of its vev will generically twist in the extra dimension.

Since the Higgs, H, is even under the orbifold Z_2, we will assume its zero mode is flat, for simplicity. Then the 4D Yukawas are determined by the 5D Yukawas and the value of the doublet wavefunctions on the branes:

$$y^u_{\alpha\beta} = Y^u_{\alpha i} Q^i_{L\beta}(0) \qquad \text{and} \qquad y^d_{\alpha\beta} = Y^d_{\alpha i} Q^i_{L\beta}(\pi) = Y^d_{\alpha i} K_{ij}(\pi) Q^j_{L\beta}(0)\,, \tag{3.3}$$

Here, $Q^i_{L\alpha}(z)$ are the three vector solutions of equation (2.4) indexed by α, with components indexed by i. The evolution matrix K_{ij} is defined in (2.6), and expresses the effect of twisting on the zero mode profiles. Because the Y's are real, and because of K has a real determinant, we have

$$\bar\theta = \mathrm{ArgDet}(K(\pi) Y^u Y^d) = 0\,. \tag{3.4}$$

But K_{ij} is complex, because of the complex vev of Φ, and so the twist induces phases into the Yukawas. More precisely, the effective mass matrix in the bulk is $M = h\langle\Phi\rangle + h^T\langle\Phi^*\rangle$. Since $[h, h^T] \neq 0$ in general, and $\langle\Phi\rangle \neq \langle\Phi^*\rangle$, the criterion (2.9) is satisfied and we should expect θ_{weak}, given in (1.4), to be large. So the strong CP problem is solved, at leading order.

3.2 Corrections

One might worry that there could be mixing between the zero modes and the higher KK modes of the fermions, which invalidate our tree level result. However, it is not hard to see that this does not happen. Because of the extra chiral fermions on the brane, the KK theory has the same number of left-handed and right-handed modes. So we can write down the effective mass matrices for the up and down type quarks as:

$$M^u = \left(\begin{array}{c|ccc} Y_{\alpha\beta}\langle H\rangle & 0 & 0 & 0 \\ \hline & \ddots & \vdots & \iddots \\ \vdots & \cdots & M^u_{KK} & \cdots \\ & \iddots & \vdots & \ddots \end{array}\right), \quad M^d = \left(\begin{array}{c|ccc} Y_{\alpha\beta}K(\pi)\langle H\rangle & 0 & 0 & 0 \\ \hline & \ddots & \vdots & \iddots \\ \vdots & \cdots & M^d_{KK} & \cdots \\ & \iddots & \vdots & \ddots \end{array}\right). \tag{3.5}$$

The entries in the top row are the mixings between the left handed KK modes and the right-handed zero modes, both of which are bulk fields. The zeros appear because in the KK basis bulk modes at different mass level are orthogonal. In addition, we cannot write brane-localized mass terms because Q_R vanishes on the brane. Also, M_u^{KK} and M_d^{KK} are hermitean, by parity, and thus have a real determinant. It follows that including the whole KK tower

$$\bar\theta = \mathrm{Arg}\ \mathrm{Det}(M^u M^d) = \mathrm{Arg}\ \mathrm{Det}(K(\pi) Y^u Y^d) = 0\,. \tag{3.6}$$

So the only relevant quantity for the strong CP problem at tree level are the zero mode Yukawas appearing in (3.3). Note that the first columns of M_u and M_d are in general non-zero and complex, because the bulk KK modes do not vanish on the branes, but they do not contribute to the determinant.

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The leading contribution to $\bar{\theta}$ in this model comes from higher dimension operators. Indeed, in 5D model building, one is confronted directly with non-renormalizability because the gauge interactions blow up near the compactification scale. For example, with a 4D gauge coupling g, one cannot take the cutoff higher than

$$\Lambda \sim \frac{24\pi^3}{Lg^2}\,, \tag{3.7}$$

where L is the compactification length. So we must worry about CP invariant higher-dimension operators which will contribute to $\bar{\theta}$ when Φ gets a vev.

As shown in figure 1, parity forbids a direct contribution to $\bar{\theta}$ in the bulk. But there are dangerous operators which may couple the bulk to the the branes. For example, consider

$$\frac{\sqrt{24\pi^3}}{\Lambda^{7/2}} i(\partial_5\Phi - \partial_5\Phi^*)F\tilde{F}\,\delta(z)\,, \tag{3.8}$$

where the factor of $\sqrt{24\pi^3}\Lambda^{-7/2}$ comes from naive dimensional analysis (NDA) in 5D[23]. This operator gives a direct contribution to θ_{QCD} once Φ gets a vev. If we assume $\langle\Phi\rangle \sim cL^{-3/2}$ then we get

$$\bar{\theta} \sim 32\pi^2 \frac{c\,g^7}{(24\pi^3)^3} \sim 7\times 10^{-7}\,, \tag{3.9}$$

where in the last expression we have taken $g \sim 1$ and $c \sim 1$.

We want to emphasize that there is *a lot* of unknown physics in this estimate. The assumption behind NDA is that the tree level contribution of higher dimension operators should be comparable to higher loop effect involving any other operators in the theory. However, it is possible that some of the fields are weakly coupled at the cutoff, as is the case for the photon in the chiral lagrangian. For example, in our case we may prefer Φ to be somewhat weakly coupled. Recall that $h\langle\Phi\rangle$ is exponentiated to get the flavor hierarchies, so we should have $h\langle\Phi\rangle \sim 5L^{-1}$. We have taken $\langle\Phi\rangle \sim L^{-3/2}$, which can be justified by observing that there is already a field with this vev, namely the radion, so we might as well have two. Then, to get the correct hierarchy Φ should couple semi-perturbatively leading to a slightly smaller estimate for $\bar{\theta}$.

There are other dangerous operators, for example,

$$\frac{4\pi\sqrt{24\pi^3}}{\Lambda^{7/2}}\left(c^1_{\alpha i}\partial_5\Phi + c^2_{\alpha i}\partial_5\Phi^*\right) H\bar{u}_{R\alpha}Q_{Li}\,\delta(z) + \text{h.c.} \tag{3.10}$$

We have used the 4D normalization for the 4D field $u_{R\alpha}$, but 5D normalization for everything else. This gives contribution to θ_{QFD} and hence $\bar{\theta}$ which is the analog of (3.9), but smaller by a factor of 8π. One might have expected that (3.8) and (3.10) should give the same contribution to $\bar{\theta}$, because the two are related by a chiral rotation. However, as discussed in the introduction and the paragraph before equation (2.9), we have already fixed a basis in which the Yukawas are real, and so we cannot perform chiral rotations anymore; the only consistent statement we can make is that the physical $\bar{\theta}$ is given by the larger of the two contributions, namely (3.9).

We find it remarkable that without any contrived assumptions, the natural size of $\bar{\theta}$ in this model is small. We can bring it down to the experimental constraint by tuning. In fact, this is what is normally done in models of spontaneous CP violation. For example, in 4D models, such as Nelson-Barr, higher-dimension operators such as (3.8) are usually ignored. This is justified, for example in [11], by taking $\langle\Phi\rangle \ll \Lambda$. Of course, it is easier to defend such an approximation in a renormalizable 4D theory than in the 5D case, but if one includes gravity or any possible UV extension of Nelson-Barr, then the tuning is just as severe.

If we are not satisfied with fine tuning, there are a number of other straightforward ways to simply prevent the leading dangerous operators from appearing at all. First, note that the dangerous terms involve couplings of Φ to brane fields. But we saw in (2.6) that even if the mass matrix twists far away from the branes, there can still be a large physical effect. We may thus suppose that Φ only has support in some region in the middle of the bulk, and then the CP problem is still solved, but the dangerous terms identically vanish. Although we do not know of a way to justify this, for example through string theory, the important point is that it turns the strong CP problem over to a completely different type of model building effort.

Another way around (3.8) and (3.10) is to prevent them with flavor symmetries. For example, suppose instead of a single complex bulk scalar, we had eight of them, Φ_{ij} transforming as an adjoint under $SU(3)_{\text{flavor}}$ [18]. This is the diagonal flavor group of the standard model, under which all the fermions transform as fundamentals. The SU(3) invariant lagrangian has diagonal Yukawas and is given by:

$$\mathcal{L} \supset \int d^5x\, \Phi_{ij}(\bar{Q}_{Li}Q_{Rj} + \bar{Q}_{Ri}Q_{Lj}) + \\ + \int d^4x\; \delta_{i\alpha} H \bar{Q}_{Li} u_{R\alpha}\big|_{z=0} + \delta_{i\alpha}\tilde{H}\bar{Q}_{Li}d_{R\alpha}\Big|_{z=\pi} + \text{h.c.} \tag{3.11}$$

We will assume that the scalar vev rotates in flavor space along the extra dimension, and so we still have a twist. This assumption may seem implausible if the scalar potential is flavor symmetric, because the rotation would cost energy, but we give several examples of how it can work in appendix B.

As before, the bulk mass matrix is hermitean everywhere (now due to the flavor structure as well as the 5D Lorentz invariance) and the effective 4D Yukawas again have a real determinant. However, the higher order corrections are now modified to accommodate the flavor symmetry. In particular, the operator (3.8) is now forbidden. The leading contribution to θ_{QCD} now depends on the specific realization of the twist (see appendix B), but must be at least second order in Φ. Typically, we find that the leading contribution to $\bar{\theta}$ comes form operators such as

$$\frac{24\pi^3}{\Lambda^6} i\text{Tr}(\partial_5\Phi)^2 F\tilde{F}\,\delta(z)\,, \tag{3.12}$$

which yields

$$\bar{\theta} \sim 32\pi^2 \frac{c^2\, g^{12}}{(24\pi^3)^5} \sim 10^{-12}\,. \tag{3.13}$$

This is now well out of reach of experimental precision.

Note that with the flavor symmetry, operators like (3.10), while not forbidden, no longer contribute to $\bar{\theta}$. Adding such corrections modifies the Yukawa couplings in (3.11) from $\delta_{i\alpha}$ to

$$\lambda^u_{i\alpha} = \delta_{i\alpha} + \left. \frac{c^1 \partial_5 \Phi_{i\alpha} + c^2 \partial_5 \Phi^T_{i\alpha}}{\Lambda^{3.5}} \right|_{z=0} \tag{3.14}$$

and a similar term for the down Yukawas at $z = \pi$. Even though this is a complex contribution to the Yukawa couplings, the Yukawa matrix is guaranteed to be hermitean by the flavor structure and thus has a real determinant. The argument for the reality of the determinant of the effective 4D Yukawa couplings thus remains intact.

3.3 Flavor

The purpose of this paper is to show that there is a natural solution to the strong CP problem in the context of split fermions, which is designed to address flavor. So we will now explain qualitatively how flavor and CP are naturally incorporated in the same setup.

In split fermions [17], the Yukawas are taken to be order one, and the flavor hierarchies come from small overlaps of the bulk wavefunctions. One might immediately wonder whether twisting would smooth out the hierarchies, but it was shown in [18] that this does not happen. In the current framework since the SU(2) doublets are in bulk and the singlets on the brane, it will not be overlap integrals which generate the hierarchies, but rather the exponential suppression of the zero mode wavefunctions on the branes. More precisely, since u_R and d_R are placed on opposite branes, the size of the effective 4D Yukawa, y^u and y^d, will be set by the value of the Q_L wavefunction at $z = 0$ and $z = \pi$ respectively. The intermediate values of the Q_L wavefunctions are basically irrelevant.

The exponential (or gaussian) behavior of the zero mode wavefunctions can be seen from the formal solution (2.6). By playing with the order one factors in the widths and centers of these profiles, we can achieve the desired structure in the standard model Yukawas. For example, we can arrange the profile of the first generation to be highly peaked somewhere near the center of of the extra dimension, so that its value at both branes is small yielding light u and d quarks. Similarly, the second generation could be a broader peak, while the third generation is roughly flat. By moving the centers of the profiles slightly closer to one brane than the other, we can establish mass differences within a generation, for example, between u and d. See figure 3.

Up until now, we have not really shown how the CKM mixing will occur. Essentially, mixing will be induced due to the twist that introduces a relative rotation between the mass eigenstates at $z = 0$ (up-type) and $z = \pi$ (down type). As mentioned in section 2, since this rotation is generally complex so (2.9) is satisfied, there should be an order one CKM phase. Just to make sure that there is no accidental symmetry which makes the CKM phase vanish, we have numerically solved the system in a few simple cases and verified in fact that θ_{weak} is large and $\bar{\theta}$ is zero.

We furthermore expect that the hierarchical values of the quark doublet profiles which lead to the mass hierarchy will dictate the texture of the CKM matrix. Our framework may thus be viewed as 'predictive' in the sense that the textures of both the mass hierarchy and the mixing are set only by 6 free parameters. Again, we tried plugging in a simple

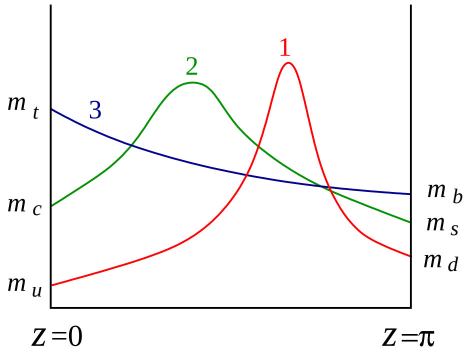

Figure 3: A cartoon showing how mass hierarchies can be generated if only the SU(2) doublets are in the bulk. $1, 2$ and 3 refer to the norms of the zero mode wavefunctions for the 3 generations of standard model quarks.

guess and numerically evaluated the result. It is not hard to get a rough quantitative agreement with the standard model; but this picture, at least qualitatively, gives the correct features.

Finally, we should also mention that the strongest constraint on flavor models usually comes from flavor-changing neutral currents (FCNCs). In extra-dimensional models, the strongest constraint comes from the exchange of KK gluons [24] which puts a lower bound on the compactification scale: $1/L \lesssim 1000$ TeV. Our estimates for $\bar{\theta}$ were basically independent of L, and so we have no problem with simply taking L as small as necessary.

4. Deconstruction

In this section we will deconstruct the twisting solution for the CP problem to obtain a purely 4D mechanism. Our initial motivation was to understand the twisting within 4D effective theory. However, as we will now show, the 4D model does not work in the same way as the 5D one. Nevertheless, we end up with a 4D model of spontaneous CP breaking which is viable but does not fall into the Nelson-Barr class.

In practice, deconstructing higher dimensional theories with fermions is surprisingly tricky. In the continuum description, one can produce a chiral theory from a vectorlike one using boundary conditions. On the lattice, an analogous mechanism is not known — deconstructing bulk fermions directly will always lead to the same number of left and right-handed modes. Thus, in the deconstructed description one needs to *start* with a chiral theory. Moreover, there is also no exact symmetry in 4D corresponding to 5D locality, so one cannot reliably confine these chiral modes to a particular site. In practice, one can simulate locality in theory space with some global symmetries, such as discrete Z_2s. and arrange a mass matrix that will concentrate the zero modes on the end cites [25]. So, in the end, it is not that difficult to create a 4D model which qualitatively matches a 5D one, but we cannot sincerely interpret it as deconstructing the 5D mechanism.

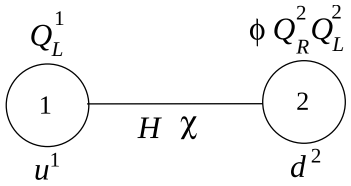

Figure 4: The matter content of our four dimensional model as distributed on the two sites.

4.1 Tree level solution

Let us construct a 2-site model using the ingredients of the 5D solution. We start by introducing the matter content of our model, which is summarized in figure 4. The fields are labeled 1 or 2 for the first or second site. As in the 5D model, we want the doublets to live in the bulk and be vectorlike, so we denote them Q_L^a and Q_R^a, $a = 1, 2$. To match the higher dimension model, right-handed bulk fermions should not have zero modes. We will achieve this by simply not including Q_R^1 into the theory, thinking of site 1 as a brane. We will think of site 2 as bulk or brane as necessary. In the 5D model the singlets, u and d, are on the boundary, so we add only u^1 and d^2. The Higgs boson, H, lives in the bulk, thus it has couplings to both sites. To complete the matter sector of the model we introduce a complex field ϕ^2 on the second site ('in the bulk') and a real link field χ that connects the two sites. These fields will play a role similar to Φ, by violating CP and propagating a twist.

Without enlarging the SM gauge group, or deconstructing it, the renormalizable couplings between Q_L^1 and d^2 (or Q_L^2 and u^1) can only be prevented through the use of the global symmetries. These symmetries serve the function of locality in the extra dimension. A convenient assignment of Z_2 charges is

	Q_L^1	Q_L^2	Q_R^2	u^1	d^2	ϕ	χ	H
Z_2^1	$-$	$+$	$+$	$-$	$+$	$+$	$-$	$+$
Z_2^2	$+$	$-$	$+$	$+$	$-$	$-$	$+$	$+$
Z_2^{12}	$+$	$+$	$-$	$+$	$+$	$-$	$-$	$+$

The most general lagrangian with this set of discrete symmetries is:

$$\begin{aligned}\mathcal{L} = \mathcal{L}_{\text{kin}} &- \left[Y_u^{ij}\tilde{H}\bar{Q}_{Li}^1 u_j^1 + Y_d^{ij} H \bar{Q}_{Li}^2 d_j^2 + h.c\right] - \\ &- \left[\left(\phi h^{ij} + \phi^* h_P^{ij}\right)\bar{Q}_{Li}^2 Q_{Rj}^2 + \text{h.c}\right] - \chi f^{ij}\bar{Q}_{Li}^1 Q_{Rj}^2 - \chi f^{ij^T}\bar{Q}_{Ri}^2 Q_{Lj}^1 - \\ &- \lambda_R\left[(\phi+\phi^*)^2 - \mu_R^2\right]^2 - \lambda_I\left[(\phi-\phi^*)^2 - \mu_I^2\right]^2 - \lambda_\chi\left(\chi^2 - \mu_\chi^2\right)^2 .\end{aligned} \tag{4.1}$$

We also assume CP is symmetry, spontaneously broken by the vev of ϕ. This forces all the Yukawa couplings to be real.

In addition we choose the Yukawa couplings of ϕ to preserve parity on site 2, that is we take $h_P = h^T$. In contrast to the 5D theory, in which parity is guaranteed by 5D Lorentz invariance, this parity cannot be taken as an exact symmetry. It is just a tree-level assumption. The essential difficulty is that parity is badly broken in the two site model, where the bulk and boundaries are indistinguishable. We will return to this issue in the next subsection.

Now, once ϕ and χ get vevs (but ignoring the Higgs for now) one linear combination of the doublets Q^1 and Q^2 will pick up a mass, and one will not. The massless combination is determined by the deconstructed equivalent of (2.4):

$$\tilde{f}^{ij} Q^1_{Lj} + M^{ij} Q^2_{Lj} = 0\,, \tag{4.2}$$

where $M^{ij} = \left(\langle\phi\rangle h^{ij} + \langle\phi^*\rangle h^{ij}_P \right)$ is the mass at site 2 which plays the role of the hermitean bulk mass, and $\tilde{f}^{ij} = f^{ij}\langle\chi\rangle$ is the real link. In the case that $[M, \tilde{f}] = 0$ we can simply choose a flavor basis so that (4.2) is diagonal. But if $[M, \tilde{f}] \neq 0$, the best we can do is choose a flavor basis so the Q's are diagonal on site one. Then, the zero modes are

$$\begin{pmatrix} \Psi_0 \\ -M^{-1}\tilde{f}\Psi_0 \end{pmatrix} . \tag{4.3}$$

So in this case, $M^{-1}\tilde{f}$ contains the information about the non-trivial twist. In particular, note that the twist matrix $M^{-1}\tilde{f}$ has a real determinant, like $K(\pi)$ in (3.3), because $\tilde{f}$ is real and M is hermitean. (As in the 5D model, we can choose an orthonormal basis without introducing a phase into the determinant of the zero mode, cf. Appendix A.)

Putting the Higgs back in, we can write the effective Yukawa couplings for the zero modes as the bare Yukawas times the value of the wavefunctions on the sites:

$$y_u^{\alpha\beta} = Y_u^{\alpha i}\Psi_0^{i\beta} \qquad \text{and} \qquad y_d^{ab} = Y_d^{\alpha i}(M^{-1}\tilde{f}\Psi_0)^{i\beta}\,. \tag{4.4}$$

This is the 4D analog of (3.3), and because the determinant of $M^{-1}\tilde{f}$ is real, $\bar{\theta} = 0$ at tree level. Also, like in the 5D case, because the Yukawas are in general complex, there will be a generically large CKM phase. In fact, the CKM phase is unsuppressed even when the massive modes are taken to be super heavy. To see this we can take the limit $|\langle\phi\rangle| \to \infty$ and $\langle\chi\rangle \to \infty$ such that the ratio is constant. The twist of the zero mode depends on $M^{-1}\tilde{f}$ which remains constant in this limit with an $O(1)$ phase. So this limit decouples the heavy states and leaves us with just the SM, but with $\bar{\theta} = 0$. Note that this is true in 5D too, where $\bar{\theta}$ and θ_{weak} in the effective theory are independent of the KK mass scale L^{-1}.

We can also see that the massive states do not introduce a contribution to $\bar{\theta}$. Since the theory is only two sites the full mass matrices for the up and down-type quarks, the parallel of (3.5), are only 6×6 and have the form

$$M^u = \begin{pmatrix} vY_u^{ij} & 0 \\ \tilde{f}^{ij} & M^{ij} \end{pmatrix} \qquad M^d = \begin{pmatrix} vY_d^{ij} & 0 \\ M^{ij} & \tilde{f}^{ij} \end{pmatrix}, \tag{4.5}$$

where v is the Higgs vev. The up and down matrices are separated and are triangular, so we can simply read off that the determinant is real. Therefore, as in the 5D model, the KK states do not affect that $\bar{\theta} = 0$ to leading order.

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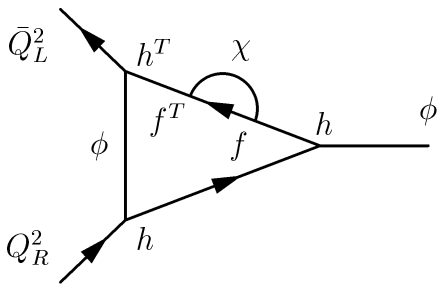

Figure 5: An example of a diagram that contributes to $\bar{\theta}$ at two loops.

4.2 Corrections

Let us return to the assumption that $h_P = h^T$, which made couplings of ϕ parity symmetric. As mentioned above, this is the analog of 5D Parity, which made M hermitean in (2.2). However, since parity is not an exact symmetry, parity violating couplings of ϕ will be generated in perturbation theory. This in turn will generate a strong CP phase. We will now estimate the minimum natural size of $h_P - h^T$ within the effective theory.

Due to the exact global symmetries of this model, the leading contribution to $h_P - h^T$ arises at two loops. For example, a correction to the $h\phi\bar{Q}_L^2 Q_R^2$ term comes from the diagram in figure 5. This and other two loop graphs give

$$\begin{aligned}\tilde{h}_P(\mu) &= h^T + \frac{1}{(16\pi^2)^2}\ln\left(\frac{\mu}{\Lambda}\right)\left(hf^Tfh^Th^T + h^Tf^Tfh^Th\right)\\ \tilde{h}(\mu) &= h + \frac{1}{(16\pi^2)^2}\ln\left(\frac{\mu}{\Lambda}\right)\left(hf^Tfhh^T + h^Tf^Tfhh\right)\,.\end{aligned} \tag{4.6}$$

The resulting P violating piece is given by

$$\tilde{h}_P(\mu)^T - \tilde{h}(\mu) \propto h\left[h\,,\,f^Tf\right]h^T + h^T\left[h\,,\,f^Tf\right]h\,. \tag{4.7}$$

We could have anticipated something like eq. (4.7) from a symmetry argument. Looking at (4.1), we see that there is a spurious SU(3) on site 1 under which f is a fundamental, and so ff^T is an adjoint, and h and h_P are adjoints too. Thus, ignoring the Yukawas, either h or ff^T alone would only break this SU(3) down to U(1)^3, and so would h and ff^T together if they could be simultaneously diagonalized. But if there were a residual U(1)^3 symmetry, the system on the first site could be factorized to three single generation subsystems which could not violate CP and P. Thus any contribution to $\bar{\theta}$ should contain the commutator factor in (4.7).

Another contribution to parity violation in the couplings of ϕ and to the strong CP phase arises from similar diagrams with d quarks in the loop. This contribution is the same as above, but has $Y_dY_d^T$ instead of ff^t. And so it is a smaller effect, because of the smallness of the down type Yukawas. Incidentally, this motivates the choice to put the d type quarks on the second site.

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The fact that (4.7) is a two loop effect gives $\bar{\theta} \sim (16\pi^2)^2 \sim 4 \times 10^{-5}$. This is quite small, but larger than current bounds. We can further suppress $\bar{\theta}$ by simply tuning the ϕ or χ Yukawas, $h, f \sim 10^{-2}$. This is a technically natural tuning because of the residual $\mathrm{U}(1)^3$ symmetry on the first or second site in the limit $f = 0$ or $h = 0$ respectively.

To summarize, we have shown that the 5D twist has a 4D analog and can be used to solve the strong CP problem. The glaring problem with the 4D model is that we must tune $h_P \approx h^T$ to achieve the equivalence of 5D parity. It is radiatively stable for these two matrices to differ by one part in 10^{-7}, but because there is no enhanced symmetry when $h_P = h^T$, this is not really any better than simply tuning $\bar{\theta}$.

It is possible to make the 4D model more technically natural, for example, by promoting $h_{ij}\phi$ to a field ϕ_{ij} transforming in the adjoint of the flavor group on site 1. Then, $\langle\phi_{ij}\rangle$ is guaranteed to be hermitean without tuning. Of course, one has to worry about contributions from the explicit flavor violation due to Y_d and f's, unless these couplings are promoted to fields as well. One could also construct more elaborate models with more sites and possibly distribute the gauge group. This will further suppresses contributions to the CP phase (for example the leading contribution appears at three loops in three site model); however, it may also require additional fine-tuning to guarantee a sufficient amount of twisting. The bottom line is that we can use this 4D setup to generate a class of solutions to the strong CP problem which are similar to, but not contained in, the Nelson-Barr class. The main drawback of these 4D models is that, in contrast to the 5D models which inspired them, they do not relate strong CP to flavor.

5. Comparison with other models

Our solution to the strong CP problem, based on twisting, is related to some other solutions and it is instructive to explore some of the similarities and differences in more detail. In our 5D model in section 3, we made essential use of both CP, which we took to be spontaneously broken, and P, which is a good symmetry in the bulk and part of 5D Lorentz invariance. The symmetry structure was shown in Figure 1. From the 4D point of view, it may seem like we have just produced another Nelson-Barr model, where the heavy fermions are just KK modes of the bulk fields. On the other hand, the use of P and CP may remind the reader of left-right (LR) symmetric models [26], which also claimed to have a natural solution to strong CP. Finally, the twisting looks a little like Hiller-Schmaltz. In this section, we clarify the distinctions between our model and these others.

5.1 Nelson-Barr

Let us start with Nelson-Barr. Nelson's original model was in the context of an SU(5) GUT, but for our purposes it is simpler to review Barr's generalization. Call the standard model fermions F, and add some vectorlike generations $R, \bar{R}$ and CP breaking scalars Φ. The global symmetries force the mass matrix to look like, in the F,R,$\bar{R}$ basis,

$$M_u \sim M_d^T \sim \begin{pmatrix} \mathbb{R}\langle H\rangle & \mathbb{C} & 0 \\ \mathbb{C} & 0 & \mathbb{R} \\ 0 & \mathbb{R} & 0 \end{pmatrix} \quad . \tag{5.1}$$

Nelson used a global U(1) to get the top zero and SO(3) flavor to get the zero in the bottom left, implementing Barr's criteria. The determinant of this matrix is real, so $\bar{\theta} = 0$ at tree level, but because of the complex numbers, θ_{weak} will in general be large.

As we have emphasized, radiative corrections to $\bar{\theta}$ from the standard model are small. If the new fields in Nelson-Barr are to preserve this situation, there needs to be a separation of scales among the CP-breaking scalar vev $\langle\Phi\rangle$, the size of the mass term $M\bar{R}R$ for the heavy fermions, and the Higgs vev $\langle H\rangle$. In particular, we need $\langle\Phi\rangle \gg M \gg \langle H\rangle$. If we ignore the tuning of the SM Higgs, this reduces to a technically natural tuning of the fermion masses M. So we can see that the virtue of Nelson-Barr is that it converts the strong CP problem into a technically natural tuning of the fermion masses M. However, there is another tuning implicit in Nelson-Barr: $\langle\Phi\rangle \ll \Lambda$, where Λ is the scale of unknown new physics. In fact, we should include non-renormalizable operators suppressed by powers Λ, such as

$$\frac{(4\pi\Phi)^n}{\Lambda^n}F\tilde{F} \quad \rightarrow \quad \bar{\theta} \sim 32\pi^2\left(4\pi\frac{\langle\Phi\rangle}{\Lambda}\right)^n \tag{5.2}$$

Here, the 4π's have been set by NDA, and the relevant power of n should be determined by the symmetries of the particular theory. But even a reasonably large n, such as $n = 4$, leads to $\langle\Phi\rangle \lesssim 10^{-5}\Lambda$.[6] This requires a tuning which is less natural than the tuning of $\bar{\theta}$, because the radiative corrections to $\bar{\theta}$, unlike $\langle\Phi\rangle$ are small. Of course, one can probably get around this tuning by adding new fields and symmetries, or by including supersymmetry, which allows $\langle\Phi\rangle$ to be naturally small.

Coming to the comparison with the current work, we see that Nelson-Barr matrix (5.1) looks something like the mass matrix we derived in section 3. That matrix, (3.5), has the schematic form:

$$M_u = \left(\begin{array}{c|c} \mathbb{R}\langle H\rangle & 0 \\ \hline \mathbb{R}\langle H\rangle & \mathbb{R} \end{array}\right), \qquad M_d = \left(\begin{array}{c|c} \mathbb{R}\langle H\rangle K & 0 \\ \hline \mathbb{R}\langle H\rangle K & \mathbb{R} \end{array}\right), \qquad \det(K) \in \mathbb{R}. \tag{5.3}$$

Recall that K is the twist matrix, which is guaranteed to have a real determinant, ultimately because of 5D Parity. In our case, the 0 arises because the heavy fermions are KK excitations of the SM zero modes, and therefore orthogonal. So not only is (5.3) organizationally different from (5.1), the underlying symmetries guaranteeing $\bar{\theta} = 0$ at leading order are not related. Thus, although our model involves spontaneous CP violation, it is really not a Nelson-Barr.

In addition, we have shown that non-renormalizable operators which contribute to $\bar{\theta}$ are naturally small. In particular, the operator (3.12) gives $\bar{\theta} \sim 10^{-12}$ without any tuning at all. In contrast, the analog (5.2) with $n = 2$ and and say $\langle\Phi\rangle = 0.01\Lambda$ leads to $\bar{\theta} \sim 1$. The root of the extra suppression in our case is the symmetry structure shown in figure 1. In particular, since CP is symmetry in the bulk and P is a symmetry on the brane, locality forces non-renormalizable terms which violate CP and P (such as $F\tilde{F}$) to involve operators like ∂_5 and $\delta(z)$. But these turn into factors of $(\Lambda L)^{-1} \ll 1$ in the low energy theory.

[6] It has been pointed out in [27] that operators like $\Phi^2\bar{R}R/\Lambda$ can also contribute to $\bar{\theta}$, forcing $\langle Phi\rangle < 10^{-10}\Lambda < 10^9$ GeV. Such a low scale for CP breaking can lead to astrophysically dangerous cosmic strings.

5.2 Left-Right models

In addition to Nelson-Barr, our model has much in common with, but is different from, left-right symmetric models [26]. Left-right models are based on the symmetry group $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$. The fermions Q_L^i and Q_R^i are fundamentals and the Higgs is a bifundamental Φ. Under parity $Q_L^i \leftrightarrow Q_R^i$ and $\Phi \leftrightarrow \Phi^\dagger$ (Φ should be thought of as a 2×2 matrix, and i is a flavor index). If we take a generic Yukawa interaction $f_{ij}\Phi\bar{Q}_iQ_j$ and add its parity and hermitean conjugate, we get

$$L \supset (f_{ij} + f_{ij}^\dagger)\Phi\bar{Q}_L^iQ_R^j + (f_{ij} + f_{ij}^\dagger)\Phi^\dagger\bar{Q}_R^iQ_L^j\,. \tag{5.4}$$

One actually needs separate Yukawas for Φ and $\sigma_2\Phi^*\sigma_2$ to have a realistic flavor model, and additional fields to break parity, but these additions are irrelevant here. To solve the strong CP problem, Φ needs to get a vev so that the effective mass matrix $M = (f + f^\dagger)\langle\Phi\rangle$ has a real determinant. There are two possibilities [28]: 1) If f is real, by CP, but $\langle\Phi\rangle$ is complex, breaking CP spontaneously, then $\det M$ will be complex, so that does not work. 2) Instead, we can take f complex but $\langle\Phi\rangle$ real, which does lead to an hermitean M; but if f is complex, CP is not a symmetry and then there is no reason to expect $\langle\Phi\rangle$ to be real. This is not to say that left-right models cannot solve strong CP (for example, there are natural left-right solutions which invoke supersymmetry [30]), only that $\bar{\theta}$ is not zero automatically.

In contrast, recall that our Yukawa interactions are of the form (2.1)

$$L \supset (f_{ij}\Phi + f_{ij}^\dagger\Phi^\star)\bar{Q}_L^iQ_R^j + (f_{ij}\Phi + f_{ij}^\dagger\Phi^\star)\bar{Q}_R^iQ_L^j\,. \tag{5.5}$$

So if f is real, but $\langle\Phi\rangle$ complex, which is natural, then the effective mass matrix will automatically be hermitean, and so the determinant is real. The difference is that our Φ is a singlet and not a bifundamental and so the couplings in (5.5) are allowed.

5.3 Hiller-Schmaltz

Finally, it is insightful to observe a formal similarity between our model and that of Hiller and Schmaltz [31, 32]. Hiller-Schmaltz is a supersymmetric model, in which CP is spontaneously broken at a much higher scale than SUSY. Due to non-renormalization theorems in SUSY, θ_{QCD} does not get renormalized. All the CP violation in the low energy theory enters through finite contributions to wavefunction renormalization factors Z_i. Because of the reality of the Kahler potential these factors come through hermitean matrices with real determinants. Thus they contribute to a CKM phase but not to θ_{QFD}, and so $\bar{\theta}$ remains zero.

To see how this relates to our model, recall that in our case, the CP violation comes in through z-dependence of the bulk fermion masses. If we preform a z-dependent flavor rotation to diagonalize these masses, undoing the twist, the CP violation appears in z-dependent kinetic terms. As in Hiller-Schmaltz, the coefficient of the kinetic terms are hermitean, and so the kinetic terms generate a CKM phase, but no $\bar{\theta}$. Actually, the analogy is not quite so clean, because the coefficient of the kinetic terms in the low energy theory will involve the twist matrix K (2.7), which is not hermitean, but still has a real determinant. And of course, in our model the phase rotation contributing to CP is a classical effect, while in Hiller-Schmaltz it comes from loops. But still, it is entertaining

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to consider the similarity between the two models, especially in light of AdS/CFT, which also relates 4D RG flows to moving along an extra dimension. Finally, we should comment that both Hiller-Schmaltz and our model solve the strong CP problem within existing frameworks, SUSY and split fermions respectively, which have been established for other purposes.

6. Discussion

Whether the strong CP problem is a serious inadequacy of the standard model depends on your attitude towards fine tuning. On the one hand, setting $\bar{\theta} = 0$ is not natural in the 't Hooft sense because no symmetry is enhanced in that limit. On the other hand, $\bar{\theta}$ is stable at 10^{-14} and so it is technically natural in the "effective field theory sense;" that is, if one imposes a cutoff (say $\Lambda \sim M_{\rm Pl}$), the radiative corrections are smaller than its initial value, even without any additional symmetries. If we believe that the second type of tuning is acceptable, there is no strong CP problem, because we can just set $\bar{\theta} = 0$ at the cutoff. However, if we allow only the first type of tuning, we should attempt to make $\bar{\theta} = 0$ 't Hooft natural by imposing a symmetry, such as CP. Then the strong CP problem becomes: why is weak CP violation in the standard model so large?

Realistic models of spontaneous CP violation which answer this question often invoke many new particles and new global symmetries, and eventually distribute the tuning of $\bar{\theta}$ among tunings of Yukawa couplings. But if one accepts that there should be new physics at some cutoff, then as we showed in section 5, one must also confront the tuning of scalar vevs. Such tunings are UV-sensitive and not technically natural in any sense, and therefore worse than simply setting $\bar{\theta}$ to zero by hand. Although one should be able to avoid the tunings by adding more structure to the model, the point remains that most models in this class are designed *just* to solve the strong CP problem.

In this paper, we not only present a solution to the strong CP problem with minimal tuning, we do so in the context of flavor. We have shown that with a simple setup — a 5D orbifold with the SU(2) doublets in the bulk, the singlets on opposite branes, and a single CP breaking scalar and Higgs — $\bar{\theta}$ comes out at 10^{-7}. No additional symmetries are assumed. This estimate is based on the most general assumptions about the UV completion; the contribution from the standard model is much smaller. Moreover, with parameters of order one that shift the bulk wavefunctions, exponentially varied masses and mixing angles can be generated using split-fermions. Of course, $\bar{\theta}$ currently looks to be smaller than 10^{-10}, and so either some tuning, or a little more model building is necessary. We have suggested some ways in which this may be done.

We have also produced a 4D realization of the twisting mechanism for breaking CP. However, in order to reproduce the effect of exact parity in the bulk one must either start with a tuned model at tree level, or assume some flavor symmetries. The simplest such realization is a two site model that gives an acceptable strong CP phase once some Yukawa couplings are assumed to be small $\sim 10^{-2}$. This model bears resemblance to conventional Nelson-Barr type models, however we have shown that it is not actually in the Nelson-Barr class.

There is much more to be done. We would like to know if a flavor model based on this setup is compatible with CKM parameters without tuning. To make progress, we would need to know more about how to choose the potential for the bulk fields so that they produce acceptable fermion profiles and sufficient twisting. If we assume flavor symmetries, we would need to know how the flavor symmetries are broken to produce a twist. Also, it would be interesting to look at the cosmology of these models, in particular at baryogenesis. It was noted in [29] that this framework may satisfy some of the Sakharov conditions — it contains a scalar that may undergo a first order phase transition, and it has a new source of CP violation. So twisted split fermions potentially addresses all of the CP issues in the standard model, strong, weak, and baryogenesis, as well as the flavor puzzle.

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A. Orthonormalization

In this appendix we will address the following subtlety. The zero mode wavefunctions ψ^i, the solutions to (2.4), are orthogonal only when we integrate over the extra dimension

$$\int_0^\pi \psi^i_\alpha(z)\psi^j_\alpha(z)\,dz = \delta^{ij}\,. \tag{A.1}$$

Recall that α labels which state and i its direction in flavor space. But the solutions we used, given by (2.5) and (2.6), do not necessarily satisfy this requirement. The concern is that by taking orthonormal linear combinations, by a transformation such as

$$\psi^i_\alpha(z)\to\psi^i_\beta(z)O^\beta_\alpha \tag{A.2}$$

we might introduce a strong CP phase. We will now show that this does not happen.

The orthonormalization procedure can be done in two steps: we first multiply by a real diagonal matrix, O_1, such that all three solutions ψ^i are normalized to 1 when integrated over the extra dimension. We then multiply by a second matrix O_2 that will render the solutions orthogonal. Without loss of generality this matrix may take the form

$$O_2 = \begin{pmatrix} 1 & \sin t e^{is} & 0 \\ 0 & \cos t & 0 \\ 0 & 0 & c \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & \sin w e^{iu} \\ 0 & 1 & \cos w \sin x e^{iv} \\ 0 & 0 & \cos w \cos x \end{pmatrix} \tag{A.3}$$

where the first matrix in the product makes ψ^2 orthogonal to ψ^1, and the second makes ψ^3 orthogonal to ψ^1 and the new ψ^2. Because both O_1 and O_2 have real determinants, the orthonormalization procedure does not introduce a phase into the determinant of the profile matrix, and thus does not damage our solution to the strong CP problem.

B. Twisting with Symmetries

In section 3.3 we mentioned that it may be useful for the bulk scalar to be an adjoint of the flavor group. We then assumed that the vev of the bulk scalar could be twisted. However, since we impose a flavor symmetry, the bulk potential for the scalar must be flavor invariant. What causes the adjoint to pick a different direction in flavor space in different locations? One would expect that a single adjoint would tend to be aligned, but also that twisting along the flat direction costs little energy. In this appendix we show some examples where a twist generically occurs.

One possibility is to add more adjoints with a non-trivial potentials such that they tend to increase at different rates. For example, with two adjoints $\Phi_{1,2}$, if the potential contains a term such as

$$V_{12} = \left[\mathrm{Tr}(\Phi^1\Phi^2)\right]^2 \,, \tag{B.1}$$

then the adjoints will prefer to point in perpendicular directions in SU(3) flavor space. If they point in commuting directions, of which there are two in SU(3), no twisting will be induced. But if we add a third adjoint with similar couplings then it cannot choose an additional orthogonal direction which is diagonal.

Now, the bulk fermions will couple to an arbitrary combination of these adjoints such that the effective mass is

$$M_{ij}(z) = \sum_A c_A \Phi^A_{ij}(z) \,. \tag{B.2}$$

If the adjoints indeed increase at different rates and do not commute, then this mass matrix is twisted. In general, if the number of adjoints is larger than the rank of the group, a twist will be induced. It is likely that fewer adjoints are necessary, but we merely wanted to present sufficient conditions that it could be done.

A second possibility is to force a twist by violating the flavor symmetry explicitly on both boundaries. Suppose that each boundary breaks the flavor group down to an SU(2) subgroup. This breaking can be communicated to the bulk by introducing a scalar field with even orbifold parity and flavor indices, say an adjoint χ. Since it is even it cannot couple to the fermions in the bulk, and thus hardly affects the zero modes. It can, however, couple to the CP breaking adjoint Φ, for example through a term like $Tr[\chi\Phi\Phi]$. An arbitrary potential can be now written for χ on the boundaries. This will induce twisting of its vev, which will in turn twist Φ.

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